

Math 4 Honors  
Lesson 1-5: Solving Inequalities with N.L.A.

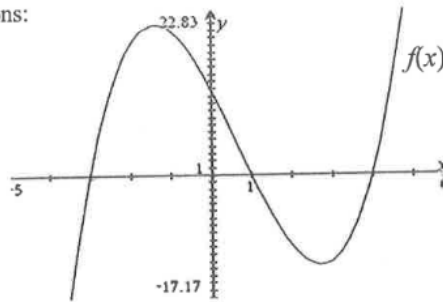
Name Heinl 2015  
Date \_\_\_\_\_

Learning Goal:

- I can use number line analysis to solve polynomial inequalities.

Use the graph of  $f(x)$  to answer the following questions:

- Identify the values for  $x$  when  $f(x) = 0$ .  
 $x = -3 \quad x = 1 \quad x = 4$
- Identify the interval(s) when  $f(x) > 0$ .  
 $(-3, 1) \cup (4, \infty)$
- Identify the interval(s) when  $f(x) < 0$ .  
 $(-\infty, -3) \cup (1, 4)$



How can we answer these questions without looking at the graph . . . . . ?

**Number Line Analysis! (NLA)**

Solve analytically:  $x^3 - 2x^2 - 11x + 12 < 0$

Factored form:  $(x+3)(x-1)(x-4) < 0$

$(x+3)$	-		+		+		+
$(x-1)$	-		-		-		+
$(x-4)$	-		-		+		+

$(-\infty, -3) \cup (1, 4)$

☆ Put critical values on # line  
 ☆ Same as # 3!

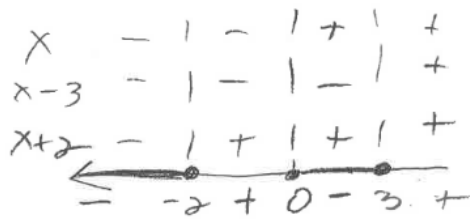
Examples: Solve the following inequalities using NLA. Check your solutions graphically.

1.  $x^3 - x^2 - 6x \leq 0$

$x(x^2 - x - 6) \leq 0$

$x(x-3)(x+2) \leq 0$

$x=0 \quad x=3 \quad x=-2$

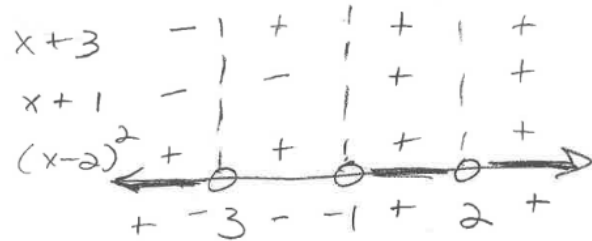


$(-\infty, -2] \cup [0, 3]$

2.  $x^4 + 4x > 9x^2 - 12$

$x^4 - 9x^2 + 4x + 12 > 0$

$x = -3 \quad x = -1 \quad x = 2$



$(-\infty, -3) \cup (-1, 2) \cup (2, \infty)$

Homework: Inequality-Solving Extravaganza

SHOW ALL WORK ON ANOTHER SHEET OF PAPER.

Use NLA to solve the following inequalities. Write your final answers in interval notation. Show all work.

1.  $(x+4)(x+1)(x-2) > 0$

2.  $x^3 - x^2 - 9x + 9 \leq 0$

3.  $-2(x-8)(x+1)^2(x-2) \leq 0$

4.  $x^3 + 4x^2 > x + 4$

5. ~~XXXXXXXXXX~~  $x^4 - 16 > 5x^3 - 20x$

6.  $x^4 - 10x^2 + 8 \geq -1$

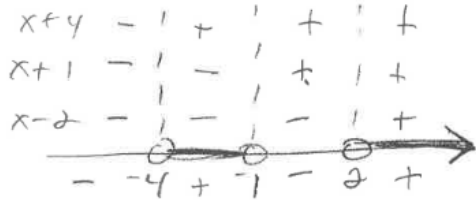
7.  $x^4 - 3x^3 - 9x^2 + 23x - 12 > 0$

8.  $-x^3 + 6x \leq 9$

### Homework Key

Lesson 1-5 HW Key

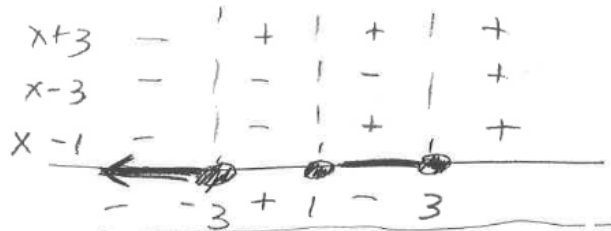
1.  $(x+4)(x+1)(x-2) > 0$



$(-4, -1) \cup (2, \infty)$

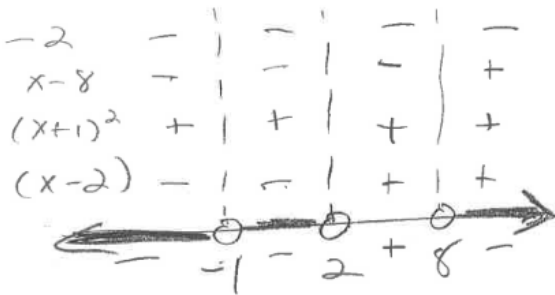
2.  $(x^3 - x^2) - (9x + 9) \leq 0$

$x^2(x-1) - 9(x+1) \leq 0$   
 $(x^2 - 9)(x-1) \leq 0$   
 $(x+3)(x-3)(x-1) \leq 0$



$(-\infty, -3] \cup [1, 3]$

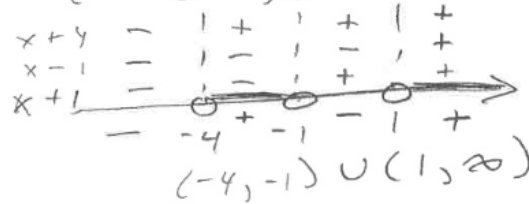
3.  $-2(x-8)(x+1)^2(x-2) < 0$



$(-\infty, -1) \cup (-1, 2) \cup (8, \infty)$

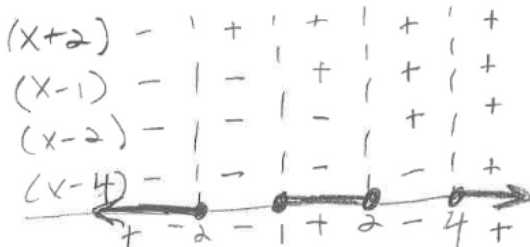
4.  $x^3 + 4x^2 > x + 4$

$(x^3 + 4x^2) - (x + 4) > 0$   
 $x^2(x+4) - (x+4) > 0$   
 $(x^2 - 1)(x+4) > 0$   
 $(x+1)(x-1)(x+4) > 0$



5.  $x^4 - 16 \geq 5x^3 - 20x$

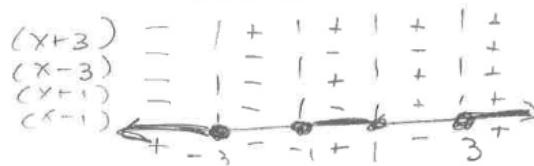
$x^4 - 5x^3 + 20x - 16 \geq 0$



$(-\infty, -2] \cup [1, 2] \cup [4, \infty)$

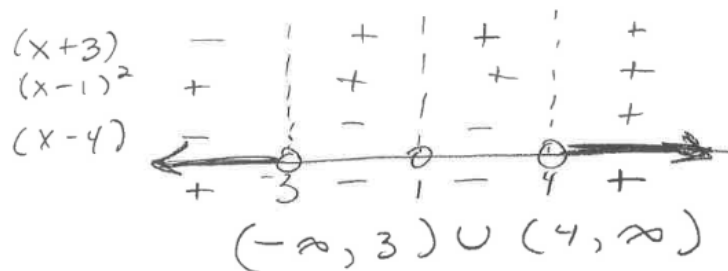
6.  $x^4 - 10x^2 + 9 \geq -1$

$x^4 - 10x^2 + 9 \geq 0$   
 $u = x^2$   
 $u^2 - 10u + 9 \geq 0$   
 $(u-9)(u-1) \geq 0$   
 $u = 9 \quad u = 1$   
 $x^2 = 9 \quad x^2 = 1$   
 $x = \pm 3 \quad x = \pm 1$



$(-\infty, -3] \cup [-1, 1] \cup [3, \infty)$

$$7. \quad x^4 - 3x^3 - 9x^2 + 23x - 12 > 0$$

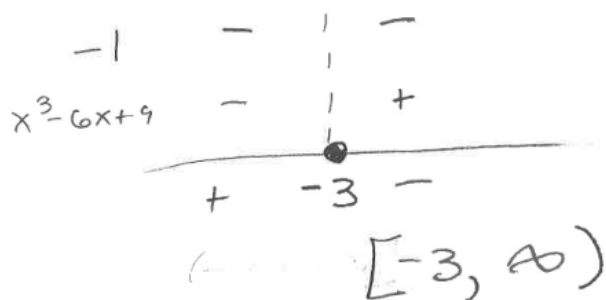


$$8. \quad -x^3 + 6x \leq 9$$

$$-x^3 + 6x - 9 \leq 0$$

$$-1(x^3 - 6x + 9) \leq 0$$

one real root:  $x = -3$



## Absolute Value Practice

$$67. \frac{x-3}{2} \geq 4 \text{ or } \frac{x-3}{2} \leq -4$$

$$x-3 \geq 8 \quad x-3 \leq -8$$

$$x \geq 11 \text{ or } x \leq -5$$

$$(-\infty, -5] \text{ or } [11, \infty)$$

$$68. \frac{-1 < 1 - \frac{2}{3}x < 1}{-1 \quad -1 \quad -1}$$

$$\frac{-2 < -\frac{2}{3}x < 0}{\cdot \frac{-3}{2} \quad \cdot \frac{-3}{2} \quad \cdot \frac{-3}{2}}$$

$$3 > x > 0$$

$$(0, 3)$$

$$69. |9-2x| - 2 < -1$$

$$\quad \quad \quad +2 \quad +2$$

$$|9-2x| < 1$$

$$\frac{-1 < 9-2x < 1}{-9 \quad -9 \quad -9}$$

$$\frac{-10 < -2x < -8}{-2 \quad -2 \quad -2}$$

$$5 > x > 4$$

$$(4, 5)$$

$$70. |x+14| + 3 > 17$$

$$|x+14| > 14$$

$$x+14 > 14 \text{ or } x+14 < -14$$

$$x > 0 \text{ or } x < -28$$

$$(-\infty, -28) \cup (0, \infty)$$

$$71. 2|x+10| \geq 9$$

$$|x+10| \geq 4.5$$

$$x+10 \geq 4.5 \text{ or } x+10 \leq -4.5$$

$$x \geq -5.5 \text{ or } x \leq -14.5$$

$$(-\infty, -14.5] \cup [-5.5, \infty)$$

$$72. 3|4-5x| \leq 9$$

$$|4-5x| \leq 3$$

$$\frac{-3 \leq 4-5x \leq 3}{-4 \quad -4 \quad -4}$$

$$\frac{-7 \leq -5x \leq -1}{-5 \quad -5 \quad -5}$$

$$\frac{7}{5} \geq x \geq \frac{1}{5}$$

$$\left[\frac{1}{5}, \frac{7}{5}\right]$$

$$125. \quad |x - 15| \leq .1$$

$$-.1 \leq x - 15 \leq .1$$

$$14.9 \leq x \leq 15.1$$

\*2.09

\$31.14

\*2.09

\$31.56

$$15 * 2.09 =$$

\$31.35 Actual Cost

$$31.35 - 31.14 = .21 \Rightarrow \text{undercharged}$$

or

$$\approx \$ .21$$

$$31.56 - 31.35 = .21 \Rightarrow \text{overcharged}$$

$$\approx \$ .21$$

128.

$$-1 \leq \frac{h - 68.5}{2.7} \leq 1$$

$$-2.7 \leq h - 68.5 \leq 2.7$$

$$+68.5 \quad +68.5 \quad +68.5$$

$$65.8 \leq h \leq 71.2$$

inches

inches

$$129. \quad -30 \leq h - 50 \leq 30$$

$$+50 \quad +50 \quad +50$$

$$20 \leq h \leq 80$$